Moving least squares approximations for surface reconstruction from point cloud data

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1 Background

A point cloud represents an unorganized set of points in $\mathbb{R}^3$, typically obtained through laser scanning of an object. Because of the limited sample density, partial information of the real object has lost in the discrete point cloud, so the object's geometry can not be completely described. In order to obtain the geometric shape of the measuring object, we must fit a proper continuous surface which preserves geometrical features of the physical entity to the point cloud. See Figure 1.

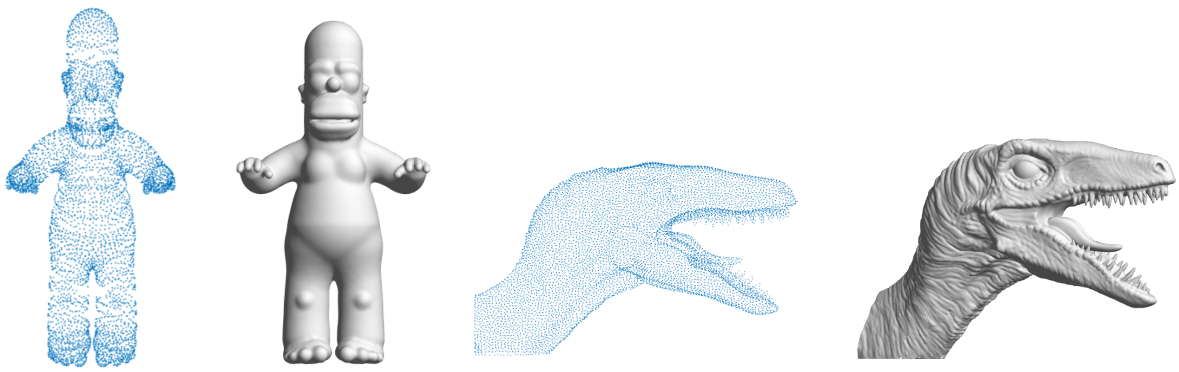

Figure 1: Point clouds and reconstructed surfaces using the method of [1].

The problem of reconstructing a surface from a point cloud has been used in a variety of applications, including computer graphics, computer aided design, medical imaging, image processing, manufacturing, and remote sensing.

2 Project description

Given a point cloud

$$X := \{\boldsymbol{x}_k = (x_k, y_k, z_k) \in \mathbb{R}^3,\ k = 1, 2, \dots, N\}$$

coming from an unknown surface Γ, the goal is to find a surface $\widehat{\Gamma}$ which is a reconstruction (approximation) of Γ. In an implicit surface approach, Γ is defined as the surface of all points $\boldsymbol{x} = (x, y, z) \in \mathbb{R}^3$ that satisfy the implicit equation

$$f(x, y, z) = 0,$$

for an unknown function f. In this case Γ is said to be the level set of f. To approximate f one can assume that f is zero at on-surface points X, and use additional off-surface points X^+ and X^- with positive and negative f values, respectively, and finally interpolate f on this $3N$ interpolation points. This approach leads to a surface reconstruction method which consists of three main steps: (1) generate off-surface points, (2) approximate f on the extended dataset, (3) compute the zero iso-surface. If the set of normals

$$\{\boldsymbol{n}_k = (n_k^x, n_k^y, n_k^z) : k = 1, \ldots, N\}$$

to Γ at points $\boldsymbol{x}_k$ are known (or can be approximated) the off-surface points can be generated by marching a small distance δ along the positive and negative directions of normals:

$$X^{\pm} = \{\boldsymbol{x}_k \pm \delta \boldsymbol{n}_k : k = 1, 2, \ldots, N\}.$$

Then we can assume that $f_X = 0$, $f_{X^+} = \epsilon$ and $f_{X^-} = -\epsilon$, for a positive ϵ, and approximate f using an appropriate technique. The zero iso-surface of f approximates Γ, hopefully well.

However, the naive method described above suffers from some disadvantages. Another approach is based on the fact that if $f : \mathbb{R}^3 \to \mathbb{R}$ defines a zero-level set Γ and $\boldsymbol{n}$ is a normal vector to Γ, then $\boldsymbol{n}$ is curl-free, i.e.,

$$\operatorname{curl}(\boldsymbol{n}) = 0.$$

The reason is clear: because $\boldsymbol{n}$ is proportional to ∇f (gradient of f) and the curl of a gradient field is zero. Consequently, we can design an algorithm to recover (approximate) the potential f, such that $\nabla f \approx \boldsymbol{n}$ at every point $\boldsymbol{x}_k$.

Other techniques, such as Poisson Surface Reconstruction, also exist, which reconstruct the surface by solving the Poisson equation [2].

All of these techniques must be paired with a stable and typically localized shattered data approximation method. In this project we focus on the *moving least squares* (MLS) approximation which is a local polynomial reproduction technique and results in an arbitrary smooth reconstruction from data values at a set of scattered data points [3]. The method is called *moving* because at every point $\boldsymbol{x}$ we must solve the a least square problem of the form

$$\operatorname*{arg\,min}_{p \in \mathbb{P}_m} \left\{ \sum_{j=1}^{N} \phi_\delta(\|\boldsymbol{x} - \boldsymbol{x}_j\|_2)(f(\boldsymbol{x}_j) - p(\boldsymbol{x}_j))^2 \right\}.$$

Here, ϕ_δ represents a radial weight function that measures the influence of neighboring points on the point $\boldsymbol{x}$. MLS is usually used in Euclidean spaces, however, we can extend it to manifolds and use the approaches outlined above for surface reconstruction.

Planed tasks are:

- learning and understanding the basics of the approximation with MLS,
- developing a numerical algorithm for surface reconstruction based on off-surface points,
- learning and understanding the interpolation with divergence-free and curl-free MLS
- developing an algorithm for surface reconstruction based on the curl-free approach.

The recommended programming environment is Matlab, although other programming languages can also be utilized.

References

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[3] D. Mirzaei, Analysis of moving least squares approximation revisited, J. Comput. Appl. Math., 282 (2015) 237–250.